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Engineering Notes

Supercruise Aircraft Range

W. H. Heiser*
U.S. Air Force Academy, Colorado 80840
and
J. D. Mattingly†
Bothell, Washington 98011

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Nomenclature

A	=	coefficient for supersonic drag due to lift
a_{STD}	=	speed of sound at sea level for the standard atmosphere, ft/s
C_D	=	vehicle drag coefficient
C_{D0}	=	vehicle zero-lift drag coefficient
C_L	=	vehicle lift coefficient
C_L^*	=	vehicle lift coefficient at maximum lift-to-drag ratio
$(C_L/C_D)^*$	=	maximum lift-to-drag ratio
C_1	=	installed engine thrust-specific-fuel-consumption coefficient, 1/hr
C_2	=	installed engine thrust-specific-fuel-consumption coefficient, 1/hr
K_1	=	coefficient for vehicle drag due to lift
K_2	=	coefficient for vehicle drag due to camber
M_{CRIT}	=	vehicle transonic drag rise Mach number
M_0	=	cruise Mach number
RF	=	Range factor, mi
$\mathrm{RF}_{\mathrm{MAX}}$	=	maximum range factor, mi
RF*	=	Range factor at maximum lift-to-drag ratio, mi
s	=	distance flown, mi
TSFC	=	installed engine thrust-specific fuel consumption, 1/hr
V	=	instantaneous vehicle speed, mi/h
W	=	instantaneous vehicle weight, lbf
W_{final}	=	vehicle weight at the end of the cruise segment, lbf
W_{initial}	=	vehicle weight at the start of the cruise segment, lbf
θ	=	ratio of static temperature at altitude to that of the sea-level standard atmosphere

I. Introduction

THIS paper briefly summarizes the evaluation of the cruise range of supersonic aircraft (e.g., bombers, fighters, transports, and UAVs) by means of familiar, reliable, straightforward methods. Although the entire analysis is based upon the material of Chapters 2 and 3 of [1], similar approaches can be found in many fundamental aircraft-design textbooks, such as [2–4].

Since the results are presented in the form of algebraic equations that are easily manipulated and transparent, readers may insert their own empirical information or rearrange them for their own purposes. Nevertheless, the algebraic equations and relevant numerical illustrations included in this paper provide food for thought about the important contemporary topic of the potential cruise range of supersonic vehicles.

II. Cruising Flight Basics

Cruise very closely approaches flight at constant altitude and speed, for which lift equals instantaneous weight and installed engine thrust equals vehicle drag. Under these conditions the incremental change in instantaneous vehicle weight is entirely due to the fuel consumed for propulsion over the incremental distance flown and is given by

$$\frac{\mathrm{d}W}{W} = -\left(\frac{C_D}{C_L}\frac{\mathrm{TSFC}}{V}\right)\mathrm{d}s = -\frac{\mathrm{d}s}{\mathrm{RF}} \tag{1}$$

where RF is the range factor (also referred to as the Breguet range factor) defined as

$$\mathrm{RF} = \frac{C_L}{C_D}\frac{V}{\mathrm{TSFC}} \tag{2}$$

The appearance of the instantaneous vehicle speed V in Eq. (2) (resulting from the conversion of incremental time to incremental distance in representing fuel consumption) encourages the belief that the range factor may not diminish with velocity. In many important situations, including those described in more detail below, RF is constant, so that Eq. (1) can be directly integrated to yield

$$\frac{W_{\mathrm{final}}}{W_{\mathrm{initial}}} = \exp\left(-\frac{s}{\mathrm{RF}}\right) \tag{3}$$

Equations (1–3) show that the range factor is ideally suited to being a system measure of merit that accounts for both aerodynamic and propulsion performance. The larger the range factor, the better.

The relationship between lift and drag is conventionally given by the drag polar:

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} \tag{4}$$

where K_1 and C_{D0} can be functions of Mach number, and K_2 usually equals zero for the symmetrical airfoils of supersonic aircraft, so that

$$C_D = K_1 C_L^2 + C_{D0} \tag{5}$$

Furthermore, the TSFC of all types of jet engines (e.g., high bypass ratio turbofans, low bypass ratio turbofans, turbojets, and turboprops) can be estimated with remarkable accuracy from correlations of the form

$$\mathrm{TSFC} = C_1 + C_2 M_0 \sqrt{\theta} \tag{6}$$

where C_1 and C_2 are constants for any type of jet engine. Equations (2), (5), and (6) may be combined to yield the desired general result, namely

$$\mathrm{RF} = \frac{C_L}{K_1 C_L^2 + C_{D0}}\frac{a_{\mathrm{STD}}}{C_1/M_0 + C_2} \tag{7}$$

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*Professor of Aeronautics, Emeritus, Department of Aeronautics. Fellow AIAA.
†Mattingly Consulting, 15101 91st Place N. E. Senior Member AIAA.

III. Subsonic Cruise

The frequently encountered case of *best subsonic cruise* serves here to illustrate the analytical approach and as a standard of comparison for the several supersonic cases that follow. In subsonic cruise every coefficient of Eq. (7) remains essentially constant until the critical or transonic drag rise Mach number M_{CRIT} is reached, beyond which C_{D0} rises rapidly (e.g., Fig 2.11 of [1]). Fortuitously, the drag polar of Eq. (5) has a maximum value of

$$\left(\frac{C_L}{C_D}\right)^* = \frac{1}{2\sqrt{C_{D0}K_1}} \tag{8}$$

when

$$C_L^* = \sqrt{C_{D0}/K_1} \tag{9}$$

The largest possible subsonic value of range factor is therefore reached when $C_L = C_L^*$ and $M_0 = M_{\mathrm{CRIT}}$, and Eq. (7) becomes

$$\mathrm{RF}_{\mathrm{MAX}} = \frac{1}{2\sqrt{C_{D0}K_1}} \frac{a_{\mathrm{STD}}}{C_1/M_{\mathrm{CRIT}} + C_2} \tag{10}$$

For example, in the case of a future fighter with a nonafterburning low bypass ratio turbofan engine [e.g., Figs. 2.10 and 2.11 and Eq. (3.55a) of [1]], $K_1 = 0.18$, $C_{D0} = 0.014$, $C_1 = 0.90$ 1/hr, $C_2 = 0.30$ 1/hr, $a_{\mathrm{STD}} = 1116$ ft/s, and $M_{\mathrm{CRIT}} = 0.80$, whence $C_L^* = 0.28$ [Eq. (9)], $(C_L/C_D)^* = 10$ [Eq. (8)], and $\mathrm{RF}_{\mathrm{MAX}} =$ 5320 mi [Eq. (10)].

IV. Supercruise

Supercruise refers to supersonic flight with a nonafterburning engine. Once M_0 exceeds that of the transonic drag rise region (beyond approximately $M_0 = 1.2$), every coefficient in Eq. (7) remains essentially constant except that $K_1 = AM_0$ for supersonic flight (see Fig. 2.10 of [1]), where A is a constant. Under these conditions, Eq. (7) becomes

$$\mathrm{RF} = \frac{C_L}{AM_0C_L^2 + C_{D0}} \frac{a_{\mathrm{STD}}}{C_1/M_0 + C_2} \tag{11}$$

Further,

$$C_L^* = \sqrt{\frac{C_{D0}}{AM_0}} \tag{12}$$

so that

$$\left(\frac{C_L}{C_D}\right)^* = \frac{1}{2\sqrt{AC_{D0}M_0}} \tag{13}$$

and

$$\mathrm{RF}^* = \frac{1}{2\sqrt{AC_{D0}}} \frac{a_{\mathrm{STD}}}{C_1/\sqrt{M_0} + C_2\sqrt{M_0}} \tag{14}$$

Equation (14) reaches the maximum value of RF when

$$M_0 = \frac{C_1}{C_2} \tag{15}$$

so that

$$\mathrm{RF}_{\mathrm{MAX}} = \frac{1}{2\sqrt{AC_{D0}}} \frac{a_{\mathrm{STD}}}{2\sqrt{C_1C_2}} \tag{16}$$

Equation (15) holds special significance for this development. It is a noteworthy feature of the governing equations that the most desirable supercruise Mach number depends only upon installed engine parameters. Further, since $C_1 \geq C_2$ for low bypass ratio turbofans and turbojets, with or without afterburning [e.g., Eqs. (3.55a), (3.55b), (3.56a), and (3.56b) of [1]], they must possess a supersonic solution for the maximum value of RF. Also, since $C_1 \leq C_2$ for high bypass ratio turbofans and turboprops [e.g., Eqs. (3.54) and (3.57) of [1]], they never possess a supersonic solution for the maximum value of RF.

As an example, in the case of a future fighter with a nonafterburning low bypass ratio turbofan engine [e.g., Figs. 2.10 and 2.11 and Eq. (3.55a) of [1]], $A = 0.18$, $C_{D0} = 0.028$, $C_1 = 0.90$ 1/hr, $C_2 = 0.30$ 1/hr, and $a_{\mathrm{STD}} = 1116$ ft/s, whence $M_0 = 3.0$ [Eq. (15)], $C_L^* = 0.23$ [Eq. (12)], $(C_L/C_D)^* = 4.1$ [Eq. (13)], and $\mathrm{RF}_{\mathrm{MAX}} = 5160$ mi [Eq. (16)]. The first important conclusion to be drawn from this analysis therefore is that the maximum supercruise range factor can be almost as large as the best subsonic cruise range factor.

The resulting equations may also be used to calculate supercruise range factors for situations other than the maximum supercruise range factor. For example, when only the supercruise Mach number differs from the case directly above, then for $M_0 = 2.0$, $C_L^* = 0.28$ [Eq. (12)], and $\mathrm{RF}^* = 5050$ mi [Eq. (14)], and for $M_0 = 4.0$, $C_L^* =$ 0.20 [Eq. (12)], and $\mathrm{RF}^* = 5100$ mi [Eq. (14)]. Furthermore, when only C_L differs from the case directly above, then for $C_L = 0.18$ and $M_0 = 3.0$, $\mathrm{RF} = 5020$ mi [Eq. (11)]. Thus, the second important conclusion to be drawn from this analysis is that the supercruise range factor is quite insensitive to flight conditions away from those of the maximum supercruise range factor of 5160 mi.

In addition, the range factor penalty due to afterburning may be evaluated by this approach. For example, for the same case of the future fighter but with an afterburning low bypass ratio turbofan engine [e.g., Figs. 2.10 and 2.11 and Eq. (3.55b) of [1]], $A = 0.18$, $C_{D0} = 0.028$, $C_1 = 1.60$ 1/hr, $C_2 = 0.27$ 1/hr, and $a_{\mathrm{STD}} =$ 1116 ft/s, whence $M_0 = 5.9$, [Eq. (15)], $C_L^* = 0.16$ [Eq. (12)], $(C_L/C_D)^* = 2.9$ [Eq. (13)], and $\mathrm{RF}_{\mathrm{MAX}} = 4080$ mi [Eq. (16)]. This is obviously an unsatisfactory situation.

The effect of improved engine thrust-specific fuel consumption also can be easily evaluated. For example, for the same case of the future fighter (e.g., Figs. 2.10 and 2.11 of [1]), but with an improved nonafterburning low bypass ratio turbofan engine with $C_1 =$ 0.75 1/hr and $C_2 = 0.3$ 1/hr, $M_0 = 2.5$ [Eq. (15)], $C_L^* = 0.25$ [Eq. (12)], $(C_L/C_D)^* = 4.5$ [Eq. (13)], and $\mathrm{RF}_{\mathrm{MAX}} = 5650$ mi [Eq. (16)]. The improved engine gives $\mathrm{RF}_{\mathrm{MAX}} = 6120$ mi [Eq. (10)] for subsonic cruise. This is an improved situation from the initial supercruise case.

Finally, the main characteristics of supercruise found in this development should also be evident in the results of more elaborate analyses. Therefore, this approach can be used to either support or test the conclusions reached by other methods or to suggest promising directions of exploration.

V. Conclusions

There is continuing interest in practical commercial and military vehicles that can supercruise great distances. Fortunately, designers often find that supercruise range is comparable to the best available subsonic range for their vehicles. The ability to supercruise on the outbound and return portions of a military mission provides special benefits because it reduces both the flight time for the crew and the exposure to any threat from the enemy. This note is intended to increase the general awareness of this situation, and to provide the reader with fundamental analytical tools that clearly and convincingly reveal this possibility and also enable and encourage independent exploration.

References

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